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Biases in Mid-Year and End-of-Year Conventions in Discounted Cash Flow Models for Corporate Valuations

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Abstract *The end-of-year convention and the mid-year convention are two of the most widely used methodologies for discounted cash flow models in economic and financial applications. This paper uses a computer simulation to investigate how well the standard conventions perform against benchmark models of known monthly cash flows. The models are developed around a corporate valuation and loss of business income framework. The findings suggest that both the end-of-year and mid-year conventions have strong biases that can materially impact the models' results.*

I. Introduction

Since the 1960s, the discounted cash flow model has been the gold standard for a wide range of economic applications including corporate valuations, capital budgeting decisions, appraising investment property, and computing loss of business income for litigation. The purpose of this paper is to provide evidence that the performance of the standard conventions used in discounting cash flow models can be significantly improved by better understanding the factors that impact the accuracy of the forecasting models. While there are many critical variables (e.g. discount rates, growth rates, seasonality of cash flows, etc.), that if estimated incorrectly, could also substantially distort the values the models compute, these problems are already well-known and documented in the literature. Thus, this paper instead focuses on the computational biases of several common conventions utilized by almost every analyst employing discounted cash flow models, the end-of-year convention and the mid-year convention.

For the interested reader, the next part of the paper (Section II) will briefly cite some of the earlier papers addressing this topic in the

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literature. It is followed by Section III describing the issues to be investigated in the current paper. Section IV provides a discussion on the data and models employed in this research. This is followed by Section V providing the results of the competing models and further analysis. Finally, Section VI summarizes the paper's conclusions and recommendations.

II. Prior Research

Lohmann and Oakford (1984) were among the first to study the computational errors introduced by mid-year and end-of-year conventions. Other researchers include Eisemann (1992), Anderson and Barber (1992), Trout (1994), Foster and Ruth (1995), and Duvall (2000). Generally, the literature shows there are significant biases depending on the particular convention employed. Unfortunately the results are limited to a very narrow range of conditions and applications.

III. Issues to be Investigated

It is not enough for an analyst to know that the mid-year convention produces higher valuations than the end-of-year convention. Rather, the magnitude and direction of the errors from the intrinsic value must also be understood along with the conditions and circumstances that make a particular method more, or less, accurate. By further scientific investigation the efficacy of competing techniques can be evaluated under conditions similar to those faced in the field to provide analysts with greater guidance. Of course, errors in forecasting cash flows or discount rates may still have a larger impact on resulting values than the selection of a particular computational method. Nevertheless, every professional should strive to reduce or minimize all sources of error or bias that may significantly understate or overstate results.

None of the papers in the literature provide guidance to the expert when discount rates are above 14%, the firm is experiencing rapid growth, there are seasonal (uneven) cash flow patterns, or the firm is a mature company with declining net cash flows. These conditions occur frequently in corporate valuation and in many other applications, including loss of business income. Thus it is important to better understand the impact various factors may have on present value determinations calculated using common conventions.

In corporate valuation, the discount rates are based on the expected return investors would require to invest in a company with similar risk. The Capital Asset Pricing Model (CAPM) is often used in determining the appropriate discount rate. Under the CAPM, the discount rate itself can be broken down into various components including the risk-free rate, the equity risk premium (market risk), and a return for company specific risk (e.g. size, industry, etc.). From 1926 to 2000, the average annual return on large publicly-traded companies was 13.0% (Ibbotson [2001]). Smaller publicly-traded companies generated higher average annual returns of 17.3% over the same time period. Since the vast majority of companies in the U.S. are small, privately-held enterprises with a perceived risk greater than those typical of publicly-traded firms in the stock market, it is not unusual for the analyst to increase the discount rate to 20% or above to reflect a higher risk premium.[1] Many venture capitalist demand returns exceeding 30% for new startups without established track records.

The most scientific way to empirically evaluate alternative methodologies is to run a simulation to test them under situations frequently encountered by economic experts in the field. For this purpose, four basic scenarios have been constructed to precisely measure both the direction and the magnitude of the bias. While these scenarios are most applicable to analysts performing corporate valuations and assessing the loss of business income, the results and conclusions are still relevant to many other applications involving discounting cash flows or income.

IV. Data and Models

With rare exceptions, businesses experience daily cash receipts and expenses. To understand the impact of the timing of the cash flows, four realistic scenarios have been developed to measure the biases of the two most commonly employed methods: end-of-year and mid-year conventions. The benchmark for comparison will be a set of known monthly cash flows. These cash flows will be evaluated over a range of positive (or negative) growth rates and incorporate several seasonal patterns. The sensitivity of the results will be tested using three discount rates (15%, 20% and 25%) which are broadly representative of the rates used in corporate finance. Finally, quarterly cash flows will also be considered to determine how far an analyst would have to go to eliminate most of the bias in the common conventions.

Four basic scenarios were developed to capture some of the most frequent patterns of corporate cash flows. They include:

[1] For example, in several recent court cases, business valuation experts used discount rates in the 15% to 25% range. In Cede & Company and Cinerama, Inc. v. Technicolor, Inc. (Delaware Supreme Court, 2005) the discount rates were 15.28% and 19.89%. In *Astleford v. IRS* (Tax Court 2008), the expert used a 25% discount rate for his present value computations. In the *Estate of William Adams v. IRS* (Tax Court 2002), the discount rates used by the two experts to assess the value of the business were 28.17% and 29.14%. When the expected growth was included to determine the capitalization rate, the rate fell to 20.5% which was accepted by the court as reasonable.

Scenario 1 is based on a young, rapidly growing technology firm with cash flows (or earnings) increasing at a steady rate of 2% or 3% per month for the next 5 years. It is not unusual for these firms to grow by 30-60% per year in the early stages of development.

Scenario 2 portrays a company operating in a stagnant market with no growth in either sales or cash flows. Examples might include a closely regulated public utility in a market without an expanding population or a company in an industry with few alternative uses for its products.

Scenario 3 represents mature companies that are losing market share to new competitors or technologies and thus are experiencing declining sales and earnings. One only has to look at Kodak and General Motors to find examples. Yet there are numerous small firms facing larger, well-financed competitors like Wal-Mart that also struggle with many of the same survival problems.

Scenario 4 represents companies with strong seasonal patterns in both sales and earnings. For instance, the retail industry realizes a disproportionately large percentage of sales and earnings in the 4th quarter of the calendar year due to the holidays. Other businesses such as marinas or hotels in recreational regions of the U.S. book most of their sales during the high season (e.g. the 2nd and 3rd quarters in the northern U.S.).

Although the scale of the cash flows in any scenario has no direct impact on the size of any methodological bias in percentage terms, the models discussed here will be based on a medium sized company with starting net cash flows of approximately $10 million a year to add realism to the discussion.

V. Results and Analysis

To illustrate the basic framework and methodology being employed in this evaluation, the results from one 5 year model will be provided in Table 1. To save space, the pattern of cash flows are shown in detail for only years 2009 and 2013 but then summarized for 2010 to 2012. This is only one of more than 100 models developed but is representative of all the others. This particular spreadsheet depicts firms in Scenario 1 with cash flows growing at a steady monthly rate of 1% throughout the evaluation period. Column 5 shows the monthly cash flows the firm will actually experience over the next five years that are to be evaluated. With known cash flows, we can focus only on the error rates from using a particular method rather than introducing forecasting errors as well. Columns 2 and 3 apply the end-of-year and mid-year conventions drawing only on the sum of the monthly cash flows on an annual basis. (Notice the mid-point of the calendar year is June 30th with 6 full months before and after.) Column 4 simply takes the annual cash flows and allocates 25% of this amount to each quarter. Given the timing of the cash flows, the present values are then computed for each methodology as reported in columns 7 through 10, at an annual discount rate of 20% per year, adjusted for the number of cash flows in the year.[2]

As reported at the bottom of Table 1 in Column 10, the monthly cash flows produce a valuation of $41,175,615 at the end of five years. This benchmark should be compared to the valuations found by the other conventions to determine both the direction and magnitude of the bias. Given the same set of information, the end-of-year convention underestimates the present worth with a value of $39,373,308 or $1,802,307 (4.38%) lower than the benchmark. In the opposite direction, the mid-year convention generates a value of $43,131,298 overestimating the actual value by $1,955,683 or 4.75%. Since it is not uncommon for opposing experts to utilize different conventions, this could produce a $3,757,990 (9.13%) difference in valuations or economic damages solely based on the method selected. In this example, if both analysts were in total agreement on the likely cash flows of a firm and the appropriate discount rate (a rare event), the computational techniques still have a very significant impact on the end result. Even acknowledging the scale of this illustration, few parties would consider a $3.7 million swing in values or damages immaterial.[3] However, it is interesting that by simply using equal quarterly cash flows (Columns 4 and 9 of Table 1) instead of annual ones in the discounting model, the accuracy improves substantially to within $280,112 or .68% of the numbers found for the benchmark. This is a more than a six-fold improvement in precision for very little additional computational burden.

Of course, the values found above could vary considerably as the facts and conditions change. To determine how sensitive the results are to changes in growth and discount rates, a series of similar models was constructed to cover a wide range of situations. The results are presented

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[2] For simplicity, all of the present values are computed with a periodic rate found by dividing the annual discount rate by the number of cash flows per year. This is the equivalent to using an Annual Percentage Rate (APR) which is intuitive, familiar to all analysts, and commonly used in many financial applications (e.g. banking). Alternatively, one could use an effective annual interest rate that forces the monthly (quarterly) compound interest rate to equal the annual interest rate. Depending on the particular situation and circumstances under evaluation, the analyst could prefer one method over the other.

[3] If judges and juries were more familiar with the underlying systematic biases, there would be a greater temptation to simply average the damage estimates of the experts using conflicting methodologies. For example, the mean of the end-of-year estimate of $39,373,308 and mid-year estimate of $43,131,298 in Table 1 (bottom), would be a value of $41,252,303. This average value would be within $76,688 (.2%) of the benchmark value of $41,175,615. Of course, there are many good reasons not to average the damage estimates of experts despite there being a tendency by judges to reach a middle-of-the-road compromise when the scientific evidence is not clear. Research studies such as this one will add to the body of knowledge to improve the accuracy of estimated values.

in Table 2 and capture situations similar to scenarios 1 through 3. In each table, the monthly growth rate in cash flows ranged from -3% to 3% to cover a wide variation in companies and economic markets. Three discount rates, 15%, 20% and 25%, were used in the present value calculations. In each table, the monthly benchmark is provided in bold print in the last column on the right. Not surprisingly, the direction of biases for both the end-of-year and mid-year conventions remained the same - that is, the end-of-year convention consistently underestimated values (a downward bias) and the mid-year convention overestimated values (an upward bias). The percentages provided right under the computed present values are the error rates from the benchmark. As shown in Table 2 (the last 3 rows of panels A and B), the firms growth rate does not have to be very high before the mid-year convention becomes less accurate than the end-of-year convention. The end-of-year convention produced smaller errors than the mid-year convention at positive growth rates for high discount rates of 20% or above. In contrast, the mid-year convention generally outperformed the end-of-year convention when discount rates were lower (e.g.15%) or when cash flows were declining (negative growth). Most interestingly, both conventions were significantly less accurate than the values found by simply using equal quarterly cash flows. The quarterly model performed much better at all discount and growth rates and was extremely accurate (error rates of less than 1%) for firms experiencing higher positive growth.

The models in Table 2 also show the results are very sensitive to the size of the discount rate employed. Generally, at larger discount rates (20%), the competing methodologies show higher errors from the benchmark than at lower discount rates (15%). However, as Trout [1994] found, the error rate can still be significant even at much lower discount rates (6%) and for relatively short time horizons.

Of course, there are a lot of other potential cash flow patterns than the ones modeled above. In particular, as described in Scenario 4 above, many firms have clear seasonal sales and cash flows (earnings) that recur every year for fundamental reasons. Unfortunately in practice, this very common situation has not been addressed in the literature. Thus, it is not clear how much impact some standard seasonal patterns will have on the size of the biases previously found. To explore this dimension further, consider two seasonal firms. One company, a retailer, produces 15% of its cash flows in quarter 1, 20% in quarter 2, 25% in quarter 3 and 40% in quarter 4 (seasonal pattern 1).[4] The second firm, a large marina located in the Midwest, generates 15% of its cash flows in quarter 1, 35% in quarter

[4] For the purposes of this discussion, assume calendar quarters. So quarter 1 covers January, February, March; quarter 2 includes April, May, June; quarter 3 covers July, August, September and quarter 4 includes October, November, and December.

2, 35% in quarter 3 and 15% in quarter 4 (seasonal pattern 2). So the question becomes: If seasonal patterns are ignored by the analyst using annual cash flows, how much of an impact will the choice of technique have on the results?

To measure how sensitive the results of discounted cash models are to firm seasonality, assume the same range of growth rates used to test scenarios 1 to 3 above. Except now we allow the actual monthly cash flows to fit the schedule described as patterns 1 and 2. For example, in seasonal pattern 1, 15% of annual cash flows of each year are allocated to the first quarter spread evenly between the months of January, February and March. The second quarter gets 20% of annual sales divided equally between the three months. To hold constant as many variables as possible, the annual cash flows were drawn directly from the constant monthly growth rates employed in the earlier models. This means that each year, the ending cash flows would be exactly the same as those originally found by using constant growth rates as reported in Tables 1 and 2. The only difference is the cash flows were allocated using a seasonal pattern within each year to determine how close the competing conventions get to the actual PV benchmark found by a monthly discounting procedure. In other words, given a known cash flow pattern, how good are the standard conventions in estimating the actual value and what is the degree of error?

The results are reported in Table 3. Additional models were constructed to capture two common seasonal patterns based on the same wide range of positive and negative growth rates. For illustration purposes, the results are reported only for the mid-range discount rate of 20%. The present values for the end-of-year and mid-year convention stay the same as those found for Table 2 (Panel B) because these methods intentionally ignore intra-year cash flows as a simplification. The alternative quarterly model also ignores seasonal patterns and simply distributes the annual cash flows equally among four quarters. Thus, only the benchmark values change as the actual monthly cash flows conform to the assumed seasonal patterns. As before, the difference in present values found by the competing methods versus the benchmark is the amount of the bias. The error is also stated as a percentage of the benchmark value. For example, in Panel A for seasonal pattern 1 in Table 3, the value for a firm growing at 2% per month for five years found by the end-of-year convention is $53,219,808. This is $1,240,300 lower (-2.28%) than the actual benchmark value of $54,460,108 which does incorporate the seasonal pattern in its computation. With the same facts, the mid-year convention performs very poorly and overstates the value by $3,839,270 or 7.05%. Once again, the very simple quarterly model with equal quarterly cash flows is still the most accurate of the methodologies with an error of only $649,926 or 1.19%. For the whole range of growth rates, the end-of-year convention understated values from $627,780 to $1,441,422 (1.92% to 3.96%) while the mid-year convention overstated values from $837,236 to $5,602,218 (5.21% to 7.45%). In general, the end-of-year convention had a smaller bias than the mid-year convention at any growth rate with seasonal pattern 1. This is not surprising given more of the cash flows are weighted toward the end of the year in quarter 4 with this seasonal pattern.

Conventional wisdom would suggest that seasonal pattern 2 (Panel B of Table 3) should favor the mid-year convention with 70% of the annual cash flows arriving during quarters 2 and 3. Surprisingly, this is not always the case. The mid-year convention produces more accurate forecasts of values only when the firm's cash flows are at a steady state or declining. For the whole range of growth rates, the end-of-year convention understated values from $938,780 to $2,915,648 (3.80% to 5.80%), while the mid-year convention overstated values from $516,236 to $4,127,992 (3.19% to 5.38%). At any positive growth rate, even under these most favorable conditions, the end-of-year convention achieves a marginally lower error rate than the mid-year convention. Once again, the quarterly model significantly outperforms both of the standard conventions regardless of the seasonal pattern actually being experienced.

VI. Conclusions and Implications

The focus of this paper was to investigate the error rates of several commonly employed conventions used in economic applications of discounted cash flow models. Through a simulation analysis employing realistic conditions often found in corporate valuation and loss of business income situations, it was shown that both the end-of-year and mid-year conventions have large biases that can materially distort the resulting values. Most problematic is that these biases are systematic, i.e. the end-of-year convention consistently understates estimated values (or economic damages) while the mid-year convention consistently overstates estimated values. As demonstrated here, even when evaluating a mid-sized firm, the differences between the two approaches can easily be several million dollars or in the range of 8-10% of the total value. This result may explain the rising popularity of the mid-year convention by plaintiff's experts while many defendants' experts still prefer the end-of-year convention. Nevertheless, every objective analyst should be concerned about the lack of computational precision and error rates of this magnitude. Few audiences relying on the results of the economic analysis are likely to fully comprehend or appreciate the distortions caused by the choice of methodology. A plaintiff should not be entitled to a large financial windfall nor should a defendant avoid paying less than the true economic value based solely on computational ease.

Seasonal patterns in cash flows, which are not unusual in business, further complicate the choice of one convention over the other, yet many analysts do not take the time to carefully examine monthly or quarterly cash flow histories before choosing one of the annual conventions. Depending of the type of seasonal pattern likely to be experienced, it can make the bias of the selected annual convention either smaller or larger. This suggests that it is important to perform due diligence by collecting at least quarterly financial data in advance of developing a model. Determining whether a business is influenced by seasonal factors and the basic pattern should not be a particularly onerous task. Interviews with management or examination of industry data can easily confirm seasonality at an early stage of the analysis.

One of the most significant findings of this paper is how robust the quarterly cash flow model is under a wide range of conditions. Simply dividing the forecasted annual cash flows into equal quarterly cash flows substantially reduces the error rates over annual conventions. Regardless of whether the actual cash flows are growing rapidly, declining, or have a strong seasonal pattern, the quarterly model generally produced results within 1% of the actual benchmark value. In contrast, the end-of-year and mid-year conventions often had error rates in the 3-7% range. This increased accuracy is fully obtainable with only a modest increase in work. This could be an important modification for an expert who a priori may not know the expected underlying cash flow pattern (e.g. a new startup company) and thus is agonizing over which convention to use.

Further exploratory research for other economic applications should be conducted in the future to better understand alternative methodologies to the end-of-year and mid-year conventions so commonly adopted today. This paper is intended to be a first step in that direction.

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Table 1 - Valuation Using Quarterly versus Annual Cash Flows
Biases Caused by Various Conventions

Parameters:

Monthly growth rate:	1%	Seasonality:	None
Annual Disc. Rate:	20%	Term:	5 years
Growth pattern:	steady increases monthly		

(1) Date	(2) EOY Annual Cash Flow	(3) MY Annual Cash Flow	(4) Quarterly Cash Flow	(5) Monthly Cash Flow	(6) Growth %	(7) EOY Annual PV @ 20%	(8) MY Annual PV @ 20%	(9) Quarterly PV @ 20%	(10) Monthly PV @ 20%
12/31/08				$833,333					
1/31/09				$841,666	1.00%				$827,869
2/28/09				$850,083	1.00%				$822,440
3/31/09			$2,668,609	$858,584	1.00%			$2,541,532	$817,047
4/30/09				$867,170	1.00%				$811,689
5/31/09				$875,841	1.00%				$806,367
6/30/09		$10,674,436	$2,668,609	$884,600	1.00%		$9,744,382	$2,420,507	$801,079
7/31/09				$893,446	1.00%				$795,826
8/31/09				$902,380	1.00%				$790,607
9/30/09			$2,668,609	$911,404	1.00%			$2,305,245	$785,423
10/31/09				$920,518	1.00%				$780,273
11/30/09				$929,723	1.00%				$775,156
12/31/09			$2,668,609	$939,020	1.00%			$2,195,471	$770,073
Annual	$10,674,436		$10,674,436	$10,674,436		$8,895,363		$9,462,755	$9,583,849
2010 Annual	$12,028,221	$12,028,221	$12,028,221	$12,028,221	1% Mo	$8,352,932	$9,150,178	$8,772,369	$8,856,322

Table 1 (continued)

(1) Date	(2) EOY Annual Cash Flow	(3) MY Annual Cash Flow	(4) Quarterly Cash Flow	(5) Monthly Cash Flow	(6) Growth %	(7) EOY Annual PV @ 20%	(8) MY Annual PV @ 20%	(9) Quarterly PV @ 20%	(10) Monthly PV @ 20%
2011 Annual	$13,553,701	$13,553,701	$13,553,701	$13,553,701	1% Mo	$7,843,577	$8,592,208	$8,132,352	$8,184,024
2012 Annual	$15,272,649	$15,272,649	$15,272,649	$15,272,649	1% Mo	$7,365,282	$8,068,263	$7,539,030	$7,562,761
1/31/13				$1,356,956	1.00%				$603,692
2/28/13				$1,370,526	1.00%				$599,733
3/31/13			$4,302,401	$1,384,231	1.00%			$1,877,123	$595,800
4/30/13				$1,398,074	1.00%				$591,894
5/31/13				$1,412,054	1.00%				$588,012
6/30/13		$17,209,604	$4,302,401	$1,426,175	1.00%		$7,576,267	$1,787,736	$584,156
7/31/13				$1,440,437	1.00%				$580,326
8/31/13				$1,454,841	1.00%				$576,521
9/30/13			$4,302,401	$1,469,389	1.00%			$1,702,606	$572,740
10/31/13				$1,484,083	1.00%				$568,984
11/30/13				$1,498,924	1.00%				$565,253
12/31/13			$4,302,401	$1,513,913	1.00%			$1,621,530	$561,547
Annual	$17,209,604		$17,209,604	$17,209,604		$6,916,154		$6,988,996	$6,988,659
Total						$39,373,308	$43,131,298	$40,895,502	$41,175,615
$Δ from Mo						**-$1,802,307**	**$1,955,683**	**-$280,112**	
%Δfrom Mo						**-4.38%**	**4.75%**	**-0.68%**	

Table 2 – Summary of Results

Panel A: 25% Discount Rate with Constant Growth or Decline Patterns

Monthly Growth Rate	Discount Rate	End-of-Year Annual Present Value	Mid-Year Annual Present Value	Quarterly Present Value	**Benchmark Monthly Present Value**
-3%	25%	$14,050,991 -7.31%	$15,709,485 3.63%	$14,850,827 -2.04%	**$15,159,545**
-2%	25%	$17,051,149 -6.69%	$19,063,764 4.33%	$17,960,843 -1.71%	**$18,272,946**
-1%	25%	$21,156,144 -6.01%	$23,653,288 5.09%	$22,199,940 -1.37%	**$22,508,062**
0%	25%	$26,892,789 -5.28%	$30,067,052 5.90%	$28,101,790 -1.02%	**$28,391,667**
1%	25%	$35,072,025 -4.52%	$39,211,716 6.75%	$36,486,332 -0.67%	**$36,731,355**
2%	25%	$46,951,605 -3.74%	$52,493,490 7.62%	$48,623,124 -0.31%	**$48,774,834**
3%	25%	$64,494,502 -2.96%	$72,107,045 8.49%	$66,490,571 0.04%	**$66,462,183**

Panel B: 20% Discount Rate with Constant Growth or Decline Patterns

Monthly Growth Rate	Discount Rate	End-of-Year Annual Present Value	Mid-Year Annual Present Value	Quarterly Present Value	**Benchmark Monthly Present Value**
-3%	20%	$15,244,540 -6.41%	$16,699,556 2.53%	$16,006,666 -1.73%	**$16,288,250**
-2%	20%	$18,636,761 -5.94%	$20,415,549 3.03%	$19,522,342 -1.47%	**$19,814,487**
-1%	20%	$23,315,718 -5.45%	$25,541,089 3.58%	$24,359,322 -1.21%	**$24,658,615**
0%	20%	$29,906,109 -4.92%	$32,760,501 4.15%	$31,155,513 -0.95%	**$31,453,788**
1%	20%	$39,373,308 -4.38%	$43,131,298 4.75%	$40,895,502 -0.68%	**$41,175,615**
2%	20%	$53,219,808 -3.83%	$58,299,378 5.35%	$55,110,034 -0.41%	**$55,338,301**
3%	20%	$73,797,807 -3.29%	$80,841,447 5.94%	$76,193,237 -0.15%	**$76,306,034**

Table 2 (continued)
Panel C: 15% Discount Rate with Constant Growth or Decline Patterns

Monthly Growth Rate	Discount Rate	End-of-Year Annual Present Value	Mid-Year Annual Present Value	Quarterly Present Value	**Benchmark Monthly Present Value**
-3%	15%	$16,638,377 -5.29%	$17,842,671 1.56%	$17,325,585 -1.38%	**$17,568,145**
-2%	15%	$20,505,347 -4.98%	$21,989,535 1.90%	$21,321,439 -1.20%	**$21,579,866**
-1%	15%	$25,883,975 -4.65%	$27,757,471 2.25%	$26,871,161 -1.01%	**$27,145,686**
0%	15%	$33,521,538 -4.30%	$35,947,844 2.62%	$34,740,497 -0.82%	**$35,028,812**
1%	15%	$44,577,361 -3.95%	$47,803,894 3.00%	$46,116,616 -0.63%	**$46,411,031**
2%	15%	$60,862,120 -3.60%	$65,267,353 3.38%	$62,852,560 -0.45%	**$63,134,678**
3%	15%	$85,219,247 -3.26%	$91,387,461 3.75%	$87,856,739 -0.26%	**$88,087,721**

Table 3 - Summary of Results with Seasonal Patterns
Panel A: 20% Discount Rate with Seasonal Pattern 1
(Q1=15%, Q2=20%, Q3=25%, Q4=40%)

Monthly Growth Rate	Discount Rate	End-of-Year Annual Present Value	Mid-Year Annual Present Value	Quarterly Present Value	**Benchmark Monthly Present Value**
-3%	20%	$15,244,540 -3.96%	$16,699,556 5.21%	$16,006,666 0.85%	**$15,872,320**
-2%	20%	$18,636,761 -3.67%	$20,415,549 5.52%	$19,522,342 0.91%	**$19,347,235**
-1%	20%	$23,315,718 -3.36%	$25,541,089 5.87%	$24,359,322 0.97%	**$24,125,126**
0%	20%	$29,906,109 -3.01%	$32,760,501 6.25%	$31,155,513 1.04%	**$30,834,175**
1%	20%	$39,373,308 -2.65%	$43,131,298 6.65%	$40,895,502 1.12%	**$40,443,656**
2%	20%	$53,219,808 -2.28%	$58,299,378 7.05%	$55,110,034 1.19%	**$54,460,108**
3%	20%	$73,797,807 -1.92%	$80,841,447 7.45%	$76,193,237 1.27%	**$75,239,229**

Panel B: 20% Disc. Rate with Seasonal Pattern 2
(Q1=15%, Q2=35%, Q3=35%, Q4=15%)

Monthly Growth Rate	Discount Rate	End-of-Year Annual Present Value	Mid-Year Annual Present Value	Quarterly Present Value	**Benchmark Monthly Present Value**
-3%	20%	$15,244,540 -5.80%	$16,699,556 3.19%	$16,006,666 -1.09%	**$16,183,320**
-2%	20%	$18,636,761 -5.52%	$20,415,549 3.49%	$19,522,342 -1.03%	**$19,726,321**
-1%	20%	$23,315,718 -5.21%	$25,541,089 3.83%	$24,359,322 -0.97%	**$24,597,830**
0%	20%	$29,906,109 -4.87%	$32,760,501 4.21%	$31,155,513 -0.90%	**$31,438,335**
1%	20%	$39,373,308 -4.52%	$43,131,298 4.60%	$40,895,502 -0.83%	**$41,236,103**
2%	20%	$53,219,808 -4.16%	$58,299,378 4.99%	$55,110,034 -0.75%	**$55,527,191**
3%	20%	$73,797,807 -3.80%	$80,841,447 5.38%	$76,193,237 -0.68%	**$76,713,455**